Undesirable Outputs and a Primal Divisia Productivity Index Based on the Directional Output Distance Function

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ABSTRACT

Despite their great popularity, the conventional Divisia productivity indexes all ignore undesirable outputs. The purpose of this study is to fill in this gap by proposing a primal Divisia-type productivity index that is valid in the presence of undesirable outputs. The new productivity index is derived by total differentiation of the directional output distance function with respect to a time trend and referred to as the Divisia–Luenberger productivity index. We also empirically compare the Divisia–Luenberger productivity index and a representative of the conventional Divisia productivity indexes — the radial-output-distance-function-based Feng and Serletis (2010) productivity index — using aggregate data on 15 OECD countries over the period 1981–2000. Our empirical results show that failure to take into account undesirable outputs not only leads to misleading rankings of countries both in terms of productivity growth and in terms of technological change, but also results in wrong conclusions concerning efficiency change.

JEL classification: D24; C43; C51; O47.
Keywords: Directional output distance function; Divisia–Luenberger productivity index; Undesirable outputs.

1 Introduction

Divisia-type productivity indexes have long enjoyed great popularity since Solow (1957) proposed his single-output dual Divisia productivity index. Jorgenson and Griliches (1967) generalized this index to a multiple-output framework, where productivity growth is calculated as the observed revenue-share-weighted output growth minus the observed cost-share-weighted input growth. Noting the restrictiveness of the assumptions of price/marginal cost proportionality and constant returns to scale underlying the Solow (1957) and Jorgenson and Griliches (1967) indexes, Caves and Christensen (1980) replaced the observed revenue shares with cost elasticities; Denny *et al.* (1981) and Fuss (1994) with cost-elasticity shares; and Diewert and Fox (2008) with markup-adjusted revenue shares (marginal revenue shares). The three resulting indexes are thus appropriate in the presence of imperfect competition, which is widely regarded to be an important feature of the economy. More recently, Feng and Serletis (2010), noting that price information is missing or distorted in many situations, derived a radial-output-distance-function-based primal Divisia productivity index. This index is dual to all the aforementioned dual Divisia indexes under different market structures, possesses certain desirable axiomatic properties, and does not require price information.

Despite the popularity of the aforementioned Divisia-type productivity indexes, a feature they all share is that they ignore undesirable (bad) outputs. In the case of the dual Divisia productivity indexes [the Solow (1957), Jorgenson and Griliches (1967), Caves and Christensen (1980), Denny *et al.* (1981), Fuss (1994), and Diewert and Fox (2008) indexes], a major reason for their incapability to deal with undesirable outputs is that the prices of undesirable outputs, needed for the construction of these indexes, are missing or distorted in most situations. In the case of the Feng and Serletis (2010) productivity index, the reason is that the radial output distance function, on which this index is based, allows only for proportional increases or contractions in outputs and thus is not suitable for situations where both expansions of desirable (good) outputs and reductions of undesirable outputs are desired. The incapability of these indexes to deal with undesirable outputs is not satisfactory in many situations. For example, in the context of global efforts to reduce greenhouse gas emissions, a desirable economic growth pattern requires both increases in desirable outputs (e.g., GDP) and reductions in CO_2 emissions. In such situations, the application of the aforementioned conventional indexes will lead to biased estimates of productivity growth.

Noting the restrictiveness of the aforementioned productivity indexes, we propose in this paper a new Divisia-type productivity index based on the directional output distance function. A major advantage of the directional output distance function is that it can simultaneously expand desirable outputs and contract undesirable outputs along a path that varies according to the direction vector adopted. Furthermore, it completely generalizes the radial Shephard input and output distance functions, providing an adequate tool to approach economic and environmental performance issues in an integrated fashion. Another advantage of this function is that, like the radial output distance function, it requires quantity information only and thus is suitable for modeling undesirable outputs, whose price information is usually missing.

In deriving the primal Divisia productivity index, we follow Solow (1957) and differentiate a transformation function (in our case, a directional output distance function) totally with respect to time. This differentiation yields an equality. On one side of the equality is a directional-output-distance-function-based productivity index, which we refer to as the Divisia–Luenberger productivity index due to the fact that the directional output distance function is a variation of Luenberger's (1992) shortage function. On the other side of the equality are two terms: a directional-output-distance-function-based technological change term plus a directional-output-distance-function-based efficiency change term. This equality constitutes our key relation for the following two reasons. First, the Divisia-Luenberger productivity index inherits the properties of the directional output distance function and allows for simultaneous expansions of good outputs and contractions of bad outputs and thus is valid for situations where bad outputs are present. Second, it allows us to decompose the Divisia-Luenberger productivity index into two terms: the directional-output-distance-function-based technological change term and the directional-output-distance-function-based efficiency change term. This decomposition is consistent with the tradition of Färe *et al.* (1994), who decompose the radial-output-distance-function-based Malmquist productivity index into a technological change component and an efficiency change component.

In addition to proposing the new Divisia-type productivity index, we are also interested in empirically examining the effects of failure to take into account bad outputs when they are present. This involves two steps: first, choosing a representative from the aforementioned conventional Divisia productivity indexes so that we can empirically compare this representative index and the Divisia–Luenberger productivity index; second, parameterizing and estimating the economic functions on which the conventional representative index and the Divisia–Luenberger productivity index are based. For the first step, we choose the Feng and Serletis (2010) productivity index as the representative of the aforementioned conventional Divisia productivity indexes, because this index is dual to all the dual Divisia productivity indexes mentioned above. Specifically, as shown by Feng and Serletis (2010), it is dual to the Jorgenson and Griliches (1967) productivity index when both the output and input markets are competitive; dual to the Diewert and Fox (2008) productivity index when market power is limited to output markets; dual to the Denny *et al.* (1981) and Fuss (1994) productivity index when market power is limited to output markets and constant returns to scale is present; and also dual to a markup-and-markdown adjusted Divisia productivity index when market power is present in both output and input markets. The generality of the Feng and Serletis (2010) productivity index makes it an ideal representative of the conventional Divisia-type productivity indexes.

Regarding the second step, for the case of the Divisia–Luenberger productivity index, a quadratic functional form is chosen for the directional output distance function since it is easy to impose the translation property with this functional form. See, for example, Chambers (2002) and Färe *et al.* (2005). For the case of the conventional Feng and Serletis (2010) productivity index, a translog functional form is chosen for the radial output distance function since it is easy to impose the linear homogeneity property with this functional form. See, for example, O'Donnell and Coelli (2005). In estimating the two parametric functions, we follow Barnett and Lee (1985), Barnett *et al.* (1991), and Barnett (2002) and stress the importance of maintained regularity conditions (curvature and monotonicity) when modeling tastes and technology. To quote Barnett (2002, p. 199), "without satisfaction of both curvature and monotonicity, the second-order conditions for optimizing behavior fail, and duality theory fails." Specifically, we explicitly produce the monotonicity and curvature conditions of the directional output distance functions and those of the radial output distance function. These conditions, by putting restrictions on the weights of the Feng and Serletis (2010) productivity index and the Divisia–Luenberger productivity index, guarantee that the two indexes are economically meaningful. In estimating the two parametric functions on which the two indexes are based, we employ a Bayesian approach. This approach has the advantage of easily incorporating the maintained regularity conditions into the estimation of the directional output distance function and the radial output distance function and thus guaranteeing the coherence between economic theory and econometric techniques.

We finally apply the above framework to the aggregate data on 15 OECD countries over the period 1981–2000. Our empirical results show that failure to take into account undesirable outputs not only leads to misleading rankings of countries both in terms of productivity growth and in terms of technological change, but also results in wrong conclusions concerning efficiency change.

The paper is organized as follows. In Section 2, we derive the directional-output-distance-function-based Divisia–Luenberger productivity index and show that it can be decomposed into a directional-output-distance-function-based technological change term plus a directional-output-distance-function-based efficiency change term. Section 3 discusses data issues. Section 4 discusses the empirical specification of the radial output distance function and the directional output distance function. In Section 5 we discuss the Bayesian stochastic frontier method of estimation while in Section 6 we present and discuss the empirical results. The last section summarizes and concludes the paper.

2 Theory

Consider a production unit that uses N inputs to produce M desirable (good) outputs and P undesirable (bad) outputs. We assume that these inputs and outputs are continuously differentiable functions of time and denote them respectively by $\boldsymbol{x}(t) \in \mathbb{R}_+^N$, $\boldsymbol{y}(t) \in \mathbb{R}_+^M$, and $\boldsymbol{b}(t) \in \mathbb{R}_+^P$, where t represents a time trend. If at any period t there exists a well-defined output set $S(t)$, then the distance of the actual period t input-output vector $\boldsymbol{z}(t) \equiv (\boldsymbol{y}(t), \boldsymbol{b}(t), \boldsymbol{x}(t))$ to the frontier of the output set is given by the value of the directional output distance function, $\vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)$, where $\boldsymbol{g} \equiv \left(\boldsymbol{g}_y, -\boldsymbol{g}_b\right)$ with $\boldsymbol{g} \in \mathbb{R}_+^M \times \mathbb{R}_+^P$ is a directional vector. Formally, $\vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)$ can be defined as in Färe *et al.* (2005) :

$$\vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right) = \max\left\{\beta : (\boldsymbol{y}(t),\boldsymbol{b}(t)) + \left(\beta\boldsymbol{g}_y, -\beta\boldsymbol{g}_b\right) \in S\left(t\right)\right\}.$$

This functions is nonnegative, non-increasing in good outputs, and non-decreasing in inputs and bad outputs. In addition, it satisfies the following translation property

$$\vec{D}_o\left(\boldsymbol{y}(t) + \kappa\boldsymbol{g}_y, \boldsymbol{b}(t) - \kappa\boldsymbol{g}_b, \boldsymbol{x}(t), t; \boldsymbol{g}\right) = \vec{D}_o\left(\boldsymbol{y}(t), \boldsymbol{b}(t), \boldsymbol{x}(t), t; \boldsymbol{g}\right) - \kappa \tag{1}$$

where κ is an arbitrary scaling factor. This property says that if $\boldsymbol{y}$ is expanded by $\kappa\boldsymbol{g}_y$ and $\boldsymbol{b}$ is contracted by $\kappa\boldsymbol{g}_b$, then the value of the resulting directional output distance function will decrease by κ.

As is well known, $\vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)$ provides an additive measure of the efficiency of the production unit in the direction of $\boldsymbol{g}$, whereby an increase in the value of $\vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)$ means a decrease in efficiency. Thus in the language of continuous time, efficiency change can be measured by $-d\vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)/dt$. By definition this total differential can be written as

$$\frac{d\vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)}{dt} = \sum_{i=1}^{M+P+N} \frac{\partial\vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)}{\partial\ln z_i}\frac{d\ln z_i\left(t\right)}{dt} + \frac{\partial\vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)}{\partial t}. \tag{2}$$

After dividing both sides of (2) by $1 + \vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)$, it is immediately clear that (2) can be rewritten as[1]

$$\begin{aligned} & -\sum_{i=1}^{M+P+N} \frac{\partial\ln\left[1 + \vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)\right]}{\partial\ln z_i}\frac{d\ln z_i\left(t\right)}{dt} \\ &= \frac{\partial\ln\left[1 + \vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)\right]}{\partial t} - \frac{d\ln\left[1 + \vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)\right]}{dt}. \end{aligned} \tag{3}$$

The monotonicity conditions of the directional output distance function imply the following restrictions on the weights of $d\ln z_i\left(t\right)/dt$: $-\partial\ln\left[1 + \vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)\right]/\partial\ln y_m \geq 0$, $-\partial\ln\left[1 + \vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)\right]/\partial\ln b_p \leq 0$, and $-\partial\ln\left[1 + \vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)\right]/\partial\ln x_n \leq 0$. With these restrictions, the left hand side of (3) has the form of a Solow-type index and is referred to in this paper as 'the Divisia-Luenberger productivity index.' For notational simplicity, we denote the Divisia-Luenberger productivity index by PG^L, where the superscript 'L' is used to indicate that this index is based on Luenberger's (1992) directional output distance

[1] Note that on the frontier where $\vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right) = 0$, $1 + \vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right) = 1$. Thus dividing both sides of (2) by $1 + \vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)$ will not change (2) on the frontier.

function. The first term on the right hand of (3) measures technological change, where technological progress (regress) is said to occur when there is an outward (inward) shift of the directional output distance frontier. The second term (including the negative sign) on the right-hand side of (3), as we have already seen, measures efficiency change (in percentage form). For simplicity, we denote the first term on the right side of (3) by TC^L and the second term (including the minus sign) on the right side of (3) by EC^L.

Expression (3) constitutes our key relation for the following two reasons. First, the Divisia-Luenberger productivity index on the left hand side of (3) allows for simultaneous expansions of good outputs and contractions of bad outputs. This can be clearly seen by expanding this index as follows

$$\begin{aligned} PG^L = & -\sum_{m=1}^{M} \frac{\partial \ln \left[1+\vec{D}_o\left(\boldsymbol{z}(t),t;\boldsymbol{g}\right)\right]}{\partial \ln y_m} \frac{d \ln y_m\left(t\right)}{dt} \\ & -\sum_{p=1}^{P} \frac{\partial \ln \left[1+\vec{D}_o\left(\boldsymbol{z}(t),t;\boldsymbol{g}\right)\right]}{\partial \ln b_p} \frac{d \ln b_p\left(t\right)}{dt} - \sum_{n=1}^{N} \frac{\partial \ln \left[1+\vec{D}_o\left(\boldsymbol{z}(t),t;\boldsymbol{g}\right)\right]}{\partial \ln x_n} \frac{d \ln x_n\left(t\right)}{dt}. \quad (4) \end{aligned}$$

The first term on the right hand side of (4) shows that an increase in a good output will lead to an increase in the value of PG^L since $-\partial \ln \left[1+\vec{D}_o\left(\boldsymbol{z}(t),t;\boldsymbol{g}\right)\right]/\partial \ln y_m \geq 0$. The second term on the right hand side of (4) shows that a reduction in a bad output will also lead to an increase in the value of PG^L since $-\partial \ln \left[1+\vec{D}_o\left(\boldsymbol{z}(t),t;\boldsymbol{g}\right)\right]/\partial \ln b_p \leq 0$. Thus the Divisia-Luenberger productivity index credits both increases in good outputs and reductions in bad outputs, and thus is suitable for situations where bad outputs are present. In contrast, the conventional Feng and Serletis (2010) productivity index allows only for proportional increases or contractions in outputs and thus is not suitable for situations where both expansions of desirable (good) outputs and reductions of undesirable outputs are desired. Moreover, the Divisia-Luenberger productivity index is also superior to the conventional dual Divisia productivity indexes in that it does not require the prices of bad outputs, which are missing or distorted in most situations.

Second, (3) provide a meaningful way to decompose the Divisia-Luenberger productivity index. The equality between PG^L and $\left(TC^L + EC^L\right)$ means that the Divisia-Luenberger productivity index can be decomposed into two components: TC^L and EC^L, with the former capturing the shift in the frontier and the latter capturing catch-up effects (i.e., the movement towards or away from the frontier). This decomposition is in line with the pioneering work by Chambers *et al.* (1996), who decompose the Luenberger productivity indicator into a technological change component and an efficiency change component.

Empirical application of (3) requires that PG^L, TC^L, and EC^L be approximated by discrete-time approximations. Continuous-time Divisia indexes are usually approximated using discrete-time Törnqvist formulae. See, for example, Star and Hall (1976), Trivedi

(1981), Balk (2005), and Diewert and Fox (2008). Following this practice, we approximate the continuous-time Divisia-Luenberger productivity index by the following discrete-time Törnqvist-type expression between periods t and $t+1$

$$PG^{L}(t+1,t) \equiv -\sum_{i=1}^{M+P+N} \frac{1}{2} \left\{ \frac{\partial \ln \left[1+\vec{D}_o(\boldsymbol{z}(t),t;\boldsymbol{g})\right]}{\partial \ln z_i} + \frac{\partial \ln \left[1+\vec{D}_o(\boldsymbol{z}(t+1),t+1;\boldsymbol{g})\right]}{\partial \ln z_i} \right\} \ln \left[\frac{z_i(t+1)}{z_i(t)}\right]. \tag{5}$$

Also, the technological change term at the right-hand side of (3) can be approximated by

$$TC^{L}(t+1,t) \equiv \frac{1}{2} \left\{ \ln \left[\frac{1+\vec{D}_o(\boldsymbol{z}(t),t+1;\boldsymbol{g})}{1+\vec{D}_o(\boldsymbol{z}(t),t;\boldsymbol{g})}\right] + \ln \left[\frac{1+\vec{D}_o(\boldsymbol{z}(t+1),t+1;\boldsymbol{g})}{1+\vec{D}_o(\boldsymbol{z}(t+1),t;\boldsymbol{g})}\right] \right\} \tag{6}$$

and the efficiency change term by

$$EC^{L}(t+1,t) \equiv -\ln \left[\frac{1+\vec{D}_o(\boldsymbol{z}(t+1),t+1;\boldsymbol{g})}{1+\vec{D}_o(\boldsymbol{z}(t),t;\boldsymbol{g})}\right]. \tag{7}$$

Combining (6) and (7), we obtain

$$TC^{L}(t+1,t) + EC^{L}(t+1,t) \equiv \frac{1}{2} \left\{ \ln \left[\frac{1+\vec{D}_o(\boldsymbol{z}(t),t;\boldsymbol{g})}{1+\vec{D}_o(\boldsymbol{z}(t+1),t;\boldsymbol{g})}\right] + \ln \left[\frac{1+\vec{D}_o(\boldsymbol{z}(t),t+1;\boldsymbol{g})}{1+\vec{D}_o(\boldsymbol{z}(t+1),t+1;\boldsymbol{g})}\right] \right\}, \tag{8}$$

or

$$\exp\left\{TC^{L}(t+1,t) + EC^{L}(t+1,t)\right\} \equiv \left\{ \left[\frac{1+\vec{D}_o(\boldsymbol{z}(t),t;\boldsymbol{g})}{1+\vec{D}_o(\boldsymbol{z}(t+1),t;\boldsymbol{g})}\right] \left[\frac{1+\vec{D}_o(\boldsymbol{z}(t),t+1;\boldsymbol{g})}{1+\vec{D}_o(\boldsymbol{z}(t+1),t+1;\boldsymbol{g})}\right] \right\}^{1/2} \tag{9}$$

the right-hand side of which is a Malmquist-type index, resembling the Malmquist–Luenberger productivity index proposed by Chung *et al.* (1997). The only difference is that the directional vector is assumed to be constant in (9), whereas it is assumed to be equal to $\boldsymbol{y}(t)$ in the Malmquist–Luenberger productivity index.

If there are no bad outputs, the corresponding part of the right-hand side of (4) will vanish and the Divisia–Luenberger productivity index will reduce to

$$PG^L = -\sum_{m=1}^{M} \frac{\partial \ln \left[1 + \vec{D}_o\left(\boldsymbol{y}(t), \boldsymbol{x}(t), t; \boldsymbol{g}_y\right)\right]}{\partial \ln y_m} \frac{d \ln y_m(t)}{dt}$$
$$- \sum_{n=1}^{N} \frac{\partial \ln \left[1 + \vec{D}_o\left(\boldsymbol{y}(t), \boldsymbol{x}(t), t; \boldsymbol{g}_y\right)\right]}{\partial \ln x_n} \frac{d \ln x_n(t)}{dt}. \tag{10}$$

It should be noted here that it is impossible to relate (10) to the radial-output-distance-function-based Feng and Serletis (2010) productivity index, through the well-known equality [see Färe and Grosskopf (2000) and Balk *et al.* (2008)]

$$D_o(\boldsymbol{y}(t), \boldsymbol{x}(t), t) \equiv 1/\left(1 + \overrightarrow{D}_o\left(\boldsymbol{y}(t), \boldsymbol{x}(t), t; \boldsymbol{y}(t)\right)\right),$$

where $D_o(\boldsymbol{y}(t),\boldsymbol{x}(t), t)$ is the radial output distance function. This is because this equality holds only when the direction vector of the directional output distance function is equal to $\boldsymbol{y}(t)$. However, the directional vector in the Divisia–Luenberger productivity index is constant.

In these situation, efficiency of the actual input-output vector $(\boldsymbol{y}(t), \boldsymbol{x}(t))$ can alternatively be measured by $D_o(\boldsymbol{y}(t),\boldsymbol{x}(t), t)$, the radial distance to the frontier of the period t technology. As is well known, an increase in the value of $D_o(\boldsymbol{y}(t),\boldsymbol{x}(t), t)$ means an increase in efficiency, implying that efficiency change can be measured by $dD_o(\boldsymbol{y}(t),\boldsymbol{x}(t), t)/dt$ in continuous time. Evaluating this total differential yields

$$\frac{dD_o(\boldsymbol{y}(t), \boldsymbol{x}(t), t)}{dt} = \sum_{m=1}^{M} \frac{\partial D_o(\boldsymbol{y}(t), \boldsymbol{x}(t), t)}{\partial \ln y_m} \frac{d \ln y_m(t)}{dt}$$
$$+ \sum_{n=1}^{N} \frac{\partial D_o(\boldsymbol{y}(t), \boldsymbol{x}(t), t)}{\partial \ln x_n} \frac{d \ln x_n(t)}{dt} + \frac{\partial D_o(\boldsymbol{y}(t), \boldsymbol{x}(t), t)}{\partial t}. \tag{11}$$

Dividing both sides of (11) by $D_o(\boldsymbol{y}(t),\boldsymbol{x}(t), t)$ and rearranging gives

$$\sum_{m=1}^{M} \frac{\partial \ln D_o(\boldsymbol{y}(t), \boldsymbol{x}(t), t)}{\partial \ln y_m} \frac{d \ln y_m(t)}{dt} + \sum_{n=1}^{N} \frac{\partial \ln D_o(\boldsymbol{y}(t), \boldsymbol{x}(t), t)}{\partial \ln x_n} \frac{d \ln x_n(t)}{dt}$$
$$= -\frac{\partial \ln D_o(\boldsymbol{y}(t), \boldsymbol{x}(t), t)}{\partial t} + \frac{d \ln D_o(\boldsymbol{y}(t), \boldsymbol{x}(t), t)}{dt}. \tag{12}$$

The linear homogeneity property and monotonicity conditions of the radial output distance function (i.e., non-decreasing in good outputs and non-increasing in inputs) imply the following restrictions on the weights of $d \ln y_m(t)/dt$ and $d \ln x_n(t)/dt$: $\sum_{m=1}^{M} \partial \ln D_o(\boldsymbol{y}(t),\boldsymbol{x}(t), t)$-

$/\partial \ln y_m = 1$, $\partial \ln D_o(\boldsymbol{y}(t),\boldsymbol{x}(t),t)/\partial \ln y_m \geq 0$, and $\partial \ln D_o(\boldsymbol{y}(t),\boldsymbol{x}(t),t)/\partial \ln x_n \leq 0$. With these restrictions, the left hand side of (12) has the standard form of a Divisia-type index. This index is in fact the Feng and Serletis (2010) productivity index. With regards to the two terms on the right hand of (12), the first term is technological change, where technological progress (regress) is said to occur when there is an outward (inward) shift of the radial output distance frontier. The second term measures efficiency change. For notational simplicity, we denote the left side term of (12) by PG^S, where the superscript 'S' is used to indicate that the index is based on Shephard's (1970) radial output distance function. Further, we denote the first term (including the minus sign) on the right hand side of (12) by TC^S, and the second term on the right hand side of (12) by EC^S. The equality between PG^S and $\left(TC^S + EC^S\right)$ means that the Feng and Serletis (2010) productivity index can also be decomposed into a technological change component and an efficiency change component.

PG^S, EC^S, and TC^S in (12) can be approximated in the same manner as that for PG^L, EC^L, and TC^L, yielding the following expressions

$$\begin{aligned} & PG^S(t+1,t) \\ & \equiv \sum_{m=1}^{M} \frac{1}{2}\left\{\frac{\partial \ln D_o(\boldsymbol{y}(t),\boldsymbol{x}(t),t)}{\partial \ln y_m} + \frac{\partial \ln D_o(\boldsymbol{y}(t+1),\boldsymbol{x}(t+1),t+1)}{\partial \ln y_m}\right\} \ln\left[\frac{y_m(t+1)}{y_m(t)}\right] \\ & + \sum_{n=1}^{N} \frac{1}{2}\left\{\frac{\partial \ln D_o(\boldsymbol{y}(t),\boldsymbol{x}(t),t)}{\partial \ln x_n} + \frac{\partial \ln D_o(\boldsymbol{y}(t+1),\boldsymbol{x}(t+1),t+1)}{\partial \ln x_n}\right\} \ln\left[\frac{x_n(t+1)}{x_n(t)}\right] \end{aligned}$$

$$TC^S(t+1,t) \equiv -\frac{1}{2}\left\{\ln\left[\frac{D_o(\boldsymbol{y}(t),\boldsymbol{x}(t),t+1)}{D_o(\boldsymbol{y}(t),\boldsymbol{x}(t),t)}\right] + \ln\left[\frac{D_o(\boldsymbol{y}(t+1),\boldsymbol{x}(t+1),t+1)}{D_o(\boldsymbol{y}(t+1),\boldsymbol{x}(t+1),t)}\right]\right\}$$

and

$$EC^S(t+1,t) \equiv \ln\left[\frac{D_o(\boldsymbol{y}(t+1),\boldsymbol{x}(t+1),t+1)}{D_o(\boldsymbol{y}(t),\boldsymbol{x}(t),t)}\right].$$

Alternatively, the equality between $\left[TC^S(t+1,t) + EC^S(t+1,t)\right]$ and $PG^S(t+1,t)$ can be shown by assuming that the radial output distance function takes a translog functional form and applying the "Translog" Identity [see Balk (1998, p. 225)] to the radial-output-distance-function-based Malmquist productivity index. For an excellent and detailed discussion, see Balk (1998, p. 97-111).

With the Divisia–Luenberger productivity index, which allows for simultaneous expansions of good outputs and contractions of bad outputs, and the Feng and Serletis (2010) productivity index, which is valid only when good outputs are present, we can empirically examine the effects of failure to take into account bad outputs when they are present. This is what we will do next.

3 Data

The annual data used in this study are obtained from the Penn World Tables (6.2) and the U.S. Energy Information Administration. The sample covers the period 1981-2000. The OECD countries examined are: Australia, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Spain, Sweden, the United Kingdom, and the United States. This gives a total of 15 countries ($K = 15$) observed over 20 years ($T = 20$).

For the specification of outputs and inputs, two outputs (one good output and one bad output) and three inputs are included. The good output is gross domestic product (GDP) (y, measured in billions of dollars), the bad output is carbon dioxide (CO_2) emissions from the combustion of energy (b, measured in billion metric tons). The three inputs are capital stock (x_1, measured in billions of dollars), employment (x_2, measured in thousands of workers), and energy (x_3, measured in quadrillion BTU). GDP and the capital stock are measured in 1981 international prices. Employment is calculated from real GDP per worker and capital is obtained from the capital stock per worker. CO_2 emissions account for only the combustion of energy.

Our treatment of CO_2 as an output instead of an input is consistent with the Ayres and Kneese (1969) materials-balance principle, which states that the weight of all material outputs of any production process equals the weight of all material inputs. In our particular case, CO_2 emissions come from the combustion of energy (one of the three inputs specified in this paper; see the definition of our CO_2 emissions in the previous paragraph) and oxygen (a free input). Thus for the materials-balance principle to hold in our case, CO_2 emissions should be treated as an output rather than an input. For a rigorous proof as to why pollutants cannot be modelled as inputs, see Pethig (2006) and Martin (1986). In addition, it is worth mentioning that there is a long tradition in the literature of productivity and efficiency of treating gas emissions (e.g., CO_2) as outputs. Studies, which treat CO_2 or other gas emissions as outputs, include but are not limited to, Pittman (1983), Gollop and Swinand (2001), Fernandez *et al.* (2002), Jeon and Sickles (2004), and Färe *et al.* (2005).

4 Model Specification

To obtain the estimates of the Feng and Serletis (2010) productivity index and those of the Divisia–Luenberger productivity index for the sample countries, we need to parameterize the two economic functions — the radial output distance function and the directional output distance function — on which the two indexes are based.

4.1 The Translog Radial Output Distance Function

We start with the radial output distance function, where the bad output cannot be explicitly accounted for and thus is ignored. We choose to parameterize it as a translog function, since the parameters of this functional form can be easily restricted to satisfy the linear homogeneity property. See, for example, O'Donnell and Coelli (2005) and Färe *et al.* (2008). The translog radial output distance function, defined over $M = 1$ good outputs and $N = 3$ inputs, can be written as

$$\begin{aligned}\ln D_o(y, \boldsymbol{x}, t) &= a_0 + a_1 \ln y + \frac{1}{2} a_{11} (\ln y)^2 \\ &+ \sum_{n=1}^{3} b_n \ln x_n + \frac{1}{2} \sum_{n=1}^{3} \sum_{j=1}^{3} b_{nj} \ln x_n \ln x_j + \delta_\tau t + \frac{1}{2} \delta_{\tau\tau} t^2 \\ &+ \sum_{n=1}^{3} g_{n1} \ln x_n \ln y + \delta_{\tau 1} t \ln y + \sum_{n=1}^{3} \widetilde{\delta}_{\tau n} t \ln x_n \end{aligned} \tag{13}$$

where t denotes a time trend. Symmetry requires $b_{nj} = b_{jn}$. The restrictions required for homogeneity of degree one in outputs are

$$a_1 = 1;\ a_{11} = 0;\ g_{n1} = 0;\ \delta_{\tau 1} = 0. \tag{14}$$

The translog radial output distance function in (13) cannot be directly estimated since $D_o(y, \boldsymbol{x}, t)$ is not observable. To deal with this problem we will exploit the linear homogeneity property of the radial output distance function and transform (13) into an estimable regression equation in the form of a standard stochastic frontier. See, for example, Lovell *et al.* (1994) and O'Donnell and Coelli (2005). Specifically, normalizing (13) by the output gives

$$\begin{aligned}\ln D_o\left(\frac{y}{y}, \boldsymbol{x}, t\right) &= \ln\left[\frac{1}{y} D_o(y, \boldsymbol{x}, t)\right] \\ &= -\ln y + \ln\left[D_o(y, \boldsymbol{x}, t)\right] \\ &= -\ln y - \epsilon, \end{aligned} \tag{15}$$

where the first equality is obtained by the linear homogeneity of $D_o(y, \boldsymbol{x}, t)$ in outputs and $\epsilon \equiv -\ln D_o(y, \boldsymbol{x}, t) \geq 0$. It is worth noting that, by definition, ϵ is a measure of technical inefficiency, which is unobservable and non-negative. Rearranging (15) yields

$$\ln y = -\ln D_o(1, \boldsymbol{x}, t) - \epsilon. \tag{16}$$

Assuming that ϵ follows a non-negative distribution and adding an independently and identically normally distributed error term, ε, (16) can be further written as

$$\ln y = -\ln D_o(1, \boldsymbol{x}, t) - \epsilon + \varepsilon. \tag{17}$$

The above procedure thus transforms (13) into (17), an estimable equation in the form of a standard stochastic frontier model with two error terms, with one (i.e., ε) capturing statistical noise and the other (i.e., ϵ) representing inefficiency.

By expanding the first term on the right hand side of (17), the stochastic radial output distance frontier model in (17) can be written more explicitly as

$$\ln y = -a_0 - \sum_{n=1}^{3} b_n \ln x_n - \frac{1}{2}\sum_{n=1}^{3}\sum_{j=1}^{3} b_{nj} \ln x_n \ln x_j - \delta_\tau t - \frac{1}{2}\delta_{\tau\tau} t^2 - \sum_{n=1}^{3} \widetilde{\delta}_{\tau n} t \ln x_n - \epsilon + \varepsilon. \quad (18)$$

When estimating (18) below, monotonicity and curvature conditions of the radial output distance function will be imposed if they are not satisfied, so that the empirical results obtained are consistent with microeconomic theory. Specifically, monotonicity requires that $D_o(\boldsymbol{y}, \boldsymbol{x}, t)$ be non-decreasing in good outputs and non-increasing in inputs. In our particular case where there is only one single output, $\partial \ln D_o(\boldsymbol{y}, \boldsymbol{x}, t) / \partial \ln y = 1 > 0$ by construction (i.e. by the linear homogeneity property). Thus, monotonicity only requires that $D_o(\boldsymbol{y}, \boldsymbol{x}, t)$ be non-increasing in inputs. Formally, for $n = 1, 2, 3$,

$$\partial \ln D_o(\boldsymbol{y}, \boldsymbol{x}, t) / \partial \ln x_n = b_n + \sum_{j=1}^{N} b_{nj} \ln x_j + g_{n1} \ln y + \widetilde{\delta}_{\tau n} t \leq 0. \quad (19)$$

As for curvature, it requires that $D_o(\boldsymbol{y}, \boldsymbol{x}, t)$ be quasi-convex in inputs and convex in outputs [see O'Donnell and Coelli (2005)]. In our particular case, the Hessian matrix of the radial output distance function with respect to output is equal to zero [i.e., $\boldsymbol{F} = [\partial^2 D_o(\boldsymbol{y}, \boldsymbol{x}, t) / \partial^2 y] = [a_{11}] = [0]$ by (14)], implying that convexity of $D_o(\boldsymbol{y}, \boldsymbol{x}, t)$ in outputs is always satisfied. For $D_o(\boldsymbol{y}, \boldsymbol{x}, t)$ to be quasi-convex in inputs it is sufficient that all the principal minors of the bordered Hessian matrix of the radial output distance function with respect to inputs are negative. For more details, see O'Donnell and Coelli (2005).

4.2 The Quadratic Directional Output Distance Function

We now turn to the parameterization of the directional output distance function. This involves choosing a functional form for $\vec{D}_o(\boldsymbol{z}(t), t; \boldsymbol{g})$ and specifying the directional vector, $\boldsymbol{g}$. For the former, we choose a quadratic functional form since the parameters of this functional form can be easily restricted to satisfy the translation property in (1). See Chambers (2002) and Färe *et al.* (2005, 2008). For the latter, we follow Färe *et al.* (2005) and set $\boldsymbol{g} = (\boldsymbol{g}_y, -\boldsymbol{g}_b) = (1, -1)$ for the following two reasons. First, a negative value (i.e. -1) for $-\boldsymbol{g}_b$ enables the Divisia–Luenberger productivity index to credit reductions in CO_2 emissions. Second, it will facilitate the imposition of the translation property on the quadratic function. The directional output distance function, defined over $M = 1$ good outputs, $P=1$ bad output, and $N=3$ inputs, can thus be written as

$$\begin{aligned}
\vec{D}_o(\boldsymbol{z},t;1,-1) &= \vec{D}_o(y,b,\boldsymbol{x},t;1,-1) \\
&= \beta_0+\alpha_1 y+\beta_1 b+\sum_{n=1}^{3}\gamma_n x_n+\beta_\tau t \\
&+ \frac{1}{2}\alpha_{11}y^2+\frac{1}{2}\beta_{11}b^2+\frac{1}{2}\sum_{n=1}^{3}\sum_{n'=1}^{3}\gamma_{nn'}x_n x_{n'}+\frac{1}{2}\beta_{\tau\tau}t^2 \\
&+ \sum_{n=1}^{3}\delta_{n1}x_n y+\sum_{n=1}^{3}\varphi_{n1}x_n b+\phi_{11}yb+\alpha_{\tau1}ty+\beta_{\tau1}tb+\sum_{n=1}^{3}\gamma_{\tau n}tx_n, \qquad (20)
\end{aligned}$$

where symmetry is imposed by setting $\gamma_{nn'}=\gamma_{n'n}$. The translation property will be satisfied if

$$\alpha_1-\beta_1=-1;\ \alpha_{11}=\beta_{11}=\phi_{11};\ \delta_{n1}=\varphi_{n1}\ \ (n=1,2,3)\,;\text{ and }\alpha_{\tau1}=\beta_{\tau1}. \qquad (21)$$

Restrictions in (21) are obtained by substituting $\boldsymbol{g}=\left(\boldsymbol{g}_y,-\boldsymbol{g}_b\right)=(1,-1)$ into the translation property in (1) — see the Appendix for proof. Färe *et al.* (2005) use the same directional output distance function as the one in (20) except those terms containing the time trend. Correspondingly, they impose the same equality restrictions as those in (21) except $\alpha_{\tau1}=\beta_{\tau1}$.

Like the radial output distance function, the directional output distance function in (20) cannot be estimated directly since $\vec{D}_o(\boldsymbol{z}(t),t;\boldsymbol{g})$ is not observable. In what follows we will exploit the translation property to transform (20) into an estimable regression equation in the form of a standard stochastic frontier. Specifically, in our particular case where $\boldsymbol{g}=\left(\boldsymbol{g}_y,-\boldsymbol{g}_b\right)=(1,-1)$, the translation property in (1) can be rewritten as follows

$$\vec{D}_o(y+\kappa,b-\kappa,\boldsymbol{x};1,-1)=\vec{D}_o(y,b,\boldsymbol{x};1,-1)-\kappa. \qquad (22)$$

Since κ is arbitrary, for empirical implementation we choose to be observation specific and set κ equal to $-y$. Thus (22) can be written as

$$\vec{D}_o(0,b+y,\boldsymbol{x};1,-1)=\vec{D}_o(y,b,\boldsymbol{x};1,-1)+y,$$

which, after rearranging, becomes

$$\begin{aligned}
y &= \vec{D}_o(0,b+y,\boldsymbol{x};1,-1)-\vec{D}_o(y,b,\boldsymbol{x};1,-1) \\
&= \vec{D}_o(0,b+y,\boldsymbol{x};1,-1)-\zeta, \qquad (23)
\end{aligned}$$

where $\zeta\equiv\vec{D}_o(y,b,\boldsymbol{x};1,-1)\geq 0$, representing environmental and technical inefficiency. When an *iid* normal error term, ξ, is added, (23) can be further written as

$$y=\vec{D}_o(0,b+y,\boldsymbol{x};1,-1)-\zeta+\xi. \qquad (24)$$

Thus, by employing the translation property the above procedure transforms (20) into (24), an estimable equation in the form of a standard stochastic frontier model with two error terms, with one (i.e., ξ) capturing statistical noise and the other (i.e., ζ) representing inefficiency. The logic here is similar to that used in transforming the inestimable radial output distance function into an estimable stochastic frontier model. The difference is that in the previous case the linear homogeneity property of the output distance function is exploited, whereas in this case the translation property of the directional output distance function is employed.

Since $\vec{D}_o\left(0, b+y, \boldsymbol{x}; 1, -1\right)$ takes a quadratic functional form, (24) can be explicitly written as

$$\begin{aligned} y = {} & \beta_0 + \beta_1 \tilde{b} + \sum_{n=1}^{3} \gamma_n x_n + \beta_\tau t + \frac{1}{2}\beta_{11}\tilde{b}^2 + \frac{1}{2}\sum_{n=1}^{3}\sum_{n'=1}^{3}\gamma_{nn'}x_n x_{n'} \\ & + \frac{1}{2}\beta_{\tau\tau}t^2 + \sum_{n=1}^{3}\varphi_{n1}x_n\tilde{b} + \beta_{\tau 1}t\tilde{b} + \sum_{n=1}^{3}\gamma_{\tau n}t x_n - \zeta + \xi, \end{aligned} \tag{25}$$

where $\tilde{b} = b + y$.

When estimating (25) below, monotonicity and curvature conditions of the directional output distance function will be imposed if they are not satisfied. Monotonicity in this case requires that $\vec{D}_o\left(\boldsymbol{z}\left(t\right), t; \boldsymbol{g}\right)$ be non-increasing in good outputs and non-decreasing in inputs and bad outputs. Formally, it requires

$$\begin{aligned} \partial \vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)/\partial y &= \alpha_1 + \alpha_{11}y + \sum_{n=1}^{3}\delta_{n1}x_n + \phi_{11}b + \alpha_{\tau 1}t \leq 0 \\ \partial \vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)/\partial b &= \beta_1 + \beta_{11}b + \sum_{n=1}^{3}\varphi_{n1}x_n + \phi_{11}y + \beta_{\tau 1}t \geq 0 \\ \partial \vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)/\partial x_n &= \gamma_n + \sum_{n'=1}^{3}\gamma_{nn'}x_{n'} + \delta_{n1}y + \varphi_{n1}b + \gamma_{\tau n}t \geq 0. \end{aligned}$$

Curvature requires $\vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)$ be jointly concave in desirable and undesirable outputs — see Chambers (2002, p. 753) and Färe *et al.* (2005, p. 475). Formally, let $\boldsymbol{w} = [y, b]$ denote the output vector and $\boldsymbol{H}$ denote the Hessian matrix of the directional output distance function with respect to outputs, i.e., $\boldsymbol{H} = [h_{11}, h_{12}; h_{21}, h_{22}]$ where $h_{ij} = \partial^2 \vec{D}_o\left(\boldsymbol{z}(t), t; \boldsymbol{g}\right)/\partial w_i \partial w_j$ $(i, j = 1, 2)$. Concavity in good and bad outputs will then be ensured if and only if all the principal minors of $\boldsymbol{H}$ that are of odd-numbered order are nonpositive and all the principal minors that are of even-numbered order are nonnegative — see Morey (1986).

5 Bayesian Estimation

Each of the two stochastic frontier models, (18) and (25), can be rewritten more compactly, in a panel data framework, as follows

$$q_{it} = \boldsymbol{s}_{it}'\boldsymbol{\theta} - u_{it} + v_{it} \tag{26}$$

where $i = 1, \cdots, K$ indicates countries and $t = 1, \cdots, T$ indicates time. For the case of (18), $q_{it} = \ln y_{it}$; $\boldsymbol{s}_{it}$ is a vector of all the relevant variables on the right hand side of (18); $\boldsymbol{\theta}$ is the corresponding vector of coefficients (including the intercept); $u_{it} = \epsilon_{it}$; and $v_{it} = \varepsilon_{it}$. For the case of (25), $q_{it} = y_{it}$; $\boldsymbol{s}_{it}$ is a vector of all the relevant variables on the right hand side of (25); $\boldsymbol{\theta}$ is the corresponding vector of coefficients (including the intercept); $u_{it} = \zeta_{it}$; and $v_{it} = \xi_{it}$. Thus in both cases, u_{it} represents inefficiency and v_{it} represents statistical noise, formally, $v_{it} \sim iidN(0, \sigma^2)$. Further, following the common practice in the Bayesian stochastic frontier literature, we assume that the two error terms in (26) are independent of each other and also of $\boldsymbol{s}$. For notational simplicity, in what follows we let $\boldsymbol{q}$, $\boldsymbol{u}$, and $\boldsymbol{v}$ denote the matrix forms of q_{it}, u_{it}, and v_{it}, respectively. Formally, $\boldsymbol{q} = (q_{11}, ..., q_{1T}, ..., q_{K1}, ..., q_{KT})'$, $\boldsymbol{u} = (u_{11}, ..., u_{1T}, ..., u_{K1}, ..., u_{KT})'$, and $\boldsymbol{v} = (v_{11}, ..., v_{1T}, ..., v_{K1}, ..., v_{KT})'$.

The formulation of our empirical model as a random effects model is convenient for Bayesian analysis. Although equation (26) can also be formulated as a fixed effects model, we prefer a random effects model. With a fixed effects model, technical efficiency is calculated relative to the country with the smallest intercept, which may not be the most efficient one in the case of the quadratic directional output distance function. This is because this function has an additive structure and thus the magnitude of the intercept is more likely to be an indicator of economy size than an indicator of efficiency level (in percentage form). For the case of the translog radial output distance function framework, the country with the smallest intercept is the most efficient one. This is because the translog output distance function has a multiplicative structure and efficiency levels obtained within this framework are in percentage form.

We first specify priors for the parameters in (26). Following Koop and Steel (2001) and O'Donnell and Coelli (2005), we adopt the following prior for $\boldsymbol{\theta}$

$$p(\boldsymbol{\theta}) \propto I(\boldsymbol{\theta} \in R_j) \tag{27}$$

where $I(\cdot)$ is an indicator function which takes the value 1 if the argument is true and 0 otherwise, and R_j is the set of permissible parameter values when no monotonicity and curvature constraints ($j = 0$) and when both monotonicity and curvature constraints ($j = 1$) must be satisfied.

We also follow O'Donnell and Coelli (2005) and use the following prior for h

$$p(h) \propto \frac{1}{h}, \quad \text{where } h = \frac{1}{\sigma^2} > 0 \tag{28}$$

implying that h is fully determined by the likelihood function — see the conditional posterior density for h in equation (34).

For the prior of u_{it}, we choose an exponential distribution. This is mainly because van den Broeck *et al.* (1994) find that models based on this distribution are reasonably robust to changes in priors. Since the exponential distribution is a special case of the gamma distribution, the prior of u_{it} can be written as

$$p\left(u_{it}\left|\lambda^{-1}\right.\right) = f_{\text{Gamma}}\left(u_{it}\left|1, \lambda^{-1}\right.\right). \tag{29}$$

According to Fernandez *et al.* (1997), in order to obtain a proper posterior we need a proper prior for the parameter, λ. Accordingly, we use the prior

$$p(\lambda^{-1}) = f_{\text{Gamma}}(\lambda^{-1}\left|1, \rho^*\right.). \tag{30}$$

For the case of the radial output distance function, $\rho^* = -\ln \tau^*$, where τ^* is the prior mean of the technical efficiency distribution — see, for example, O'Donnell and Coelli (2005). Our best knowledge of the efficiency of OECD countries is the mean efficiency value of 83% reported by Iyer *et al.* (2008) that examines the technical efficiency for OECD countries for the period 1982-2000. To investigate the sensitivity of our results to extreme changes to τ^*, we experiment with various values of τ^* ranging from 1% to 99%. The results are always the same up to the number of digits presented in Section 6, implying that our results obtained from (18) are very robust to large changes in τ^*. For the case of the directional output distance function, $\rho^* = \omega^*$, where ω^* is the prior mean of the environmental and technical efficiency distribution. To the best of our knowledge there is no study that has reported the environmental and technical efficiency for the OECD countries over a similar period. But from (25), we see that ω^* must fall within the range of $(0\% \times \overline{y}, 100\% \times \overline{y})$, where $\overline{y}$ is the mean of y. We thus set ω^* equal to $50\% \times \overline{y}$. We also investigate the sensitivity of our results to extreme changes to ω^* by experimenting with various values of ω^* within its possible range. We find that the results are always the same as those presented in Section 6, suggesting that our results obtained from (25) are very robust to large changes in ω^*.

The likelihood function of $\boldsymbol{q}$, given $\boldsymbol{\theta}, h_v, \boldsymbol{u}$, and λ^{-1}, is

$$\begin{aligned} L\left(\boldsymbol{q}\left|\boldsymbol{\theta}, h_v, \boldsymbol{u}, \lambda^{-1}\right.\right) &= \prod_{i=1}^{K}\prod_{t=1}^{T}\left\{\sqrt{\frac{h}{2\pi}}\exp\left[-\frac{h}{2}\left(q_{it} - \boldsymbol{s}_{it}'\boldsymbol{\theta} + u_{it}\right)^2\right]\right\} \\ &\propto h^{KT/2}\exp\left[-\frac{h}{2}\boldsymbol{v}'\boldsymbol{v}\right], \end{aligned} \tag{31}$$

where $\boldsymbol{v} = (\boldsymbol{q} - \boldsymbol{s}'\boldsymbol{\theta} + \boldsymbol{u})$.

Combining the likelihood function in (31) and the priors in (27)-(30), we obtain the joint posterior density

$$f\left(\boldsymbol{\theta}, h, \boldsymbol{u}, \lambda^{-1}\left|\boldsymbol{q}\right.\right) \propto h^{KT/2-1} \exp\left[-\frac{h}{2}\boldsymbol{v}'\boldsymbol{v}\right] I\left(\boldsymbol{\theta} \in R_j\right)$$
$$\times \prod_{i=1}^{K}\prod_{t=1}^{T}\left[\lambda^{-1}\exp\left(-\lambda^{-1}u_{it}\right)\right]\exp\left(-\rho^{*}\lambda^{-1}\right). \tag{32}$$

As noted above, all the productivity, technical change, and efficiency measures are functions of $\boldsymbol{\theta}$, h, $\boldsymbol{u}$, and λ^{-1}. Let $g(\boldsymbol{\theta},h,\boldsymbol{u},\lambda^{-1})$ represent one such function. In theory, we could obtain the moments of $g(\boldsymbol{\theta},h,\boldsymbol{u},\lambda^{-1})$ from the posterior density through integration. Unfortunately, these integrals cannot be computed analytically. Therefore, we use the Gibbs sampling algorithm which draws from the joint posterior density by sampling from a series of conditional posteriors. Once draws from the joint posterior density have been obtained, any posterior measure of interest can be calculated.

The full conditional posteriors of $\boldsymbol{\theta}, h, \boldsymbol{u}$, and λ^{-1} are respectively

$$p\left(\boldsymbol{\theta}\left|\boldsymbol{q}, h, \boldsymbol{u}, \lambda^{-1}\right.\right) \propto f_{\text{Normal}}\left(\boldsymbol{\theta}\left|\boldsymbol{b}, h^{-1}\left(\boldsymbol{s}\boldsymbol{s}'\right)^{-1}\right.\right) I\left(\boldsymbol{\theta} \in R_j\right) \tag{33}$$

$$p\left(h\left|\boldsymbol{q}, \boldsymbol{\theta}, \boldsymbol{u}, \lambda^{-1}\right.\right) \propto f_{\text{Gamma}}\left(h\left|\frac{KT}{2}, \frac{1}{2}\boldsymbol{v}'\boldsymbol{v}\right.\right) \tag{34}$$

$$p\left(\boldsymbol{u}\left|\boldsymbol{q}, \boldsymbol{\theta}, h, \lambda^{-1}\right.\right) \propto f_{\text{Normal}}\left(\boldsymbol{u}\left|\boldsymbol{s}'\boldsymbol{\theta} - \boldsymbol{q} - (h\lambda)^{-1}\boldsymbol{\iota}_{KT}, h^{-1}\mathbf{I}_{KT}\right.\right) \times \prod_{i=1}^{K}\prod_{t=1}^{T} I\left(\boldsymbol{u}_{it} \geq 0\right) \tag{35}$$

$$p\left(\lambda^{-1}\left|\boldsymbol{q}, \boldsymbol{\theta}, h, \boldsymbol{u}\right.\right) \propto f_{\text{Gamma}}\left(\lambda^{-1}\left|KT+1, \boldsymbol{u}'\boldsymbol{\iota}_{KT} + \rho^{*}\right.\right) \tag{36}$$

where $\boldsymbol{b} = \left(\boldsymbol{s}\boldsymbol{s}'\right)^{-1}\boldsymbol{s}[\boldsymbol{q} + \boldsymbol{u}]$, $\mathbf{I}_{KT}$ is a $KT \times KT$ identity matrix, and $\boldsymbol{\iota}_{KT}$ is a $KT \times 1$ vector of ones.

6 Empirical Results

We estimate the radial output distance function (18) and the directional output distance function (25) separately using the Bayesian procedure outlined in Section 5. As discussed in Section 4, we pay particular attention to the monotonicity and curvature conditions of the radial output distance function and those of the directional output distance function, so that the empirical results obtained are consistent with microeconomic theory. We first estimate each of these two models without imposing monotonicity and curvature conditions. However, these conditions are violated for both models, whether they are evaluated at posterior means or using 95% credible intervals. Since regularity is not attained for the unconstrained models, we reestimate (18) and (25) separately with monotonicity and curvature conditions imposed.

The estimated parameters, standard deviations, and 95% credible intervals (defined by 2.5 and 97.5 percentiles) from the two regularity-constrained models are reported in Tables 1.1 and 1.2 respectively. For the radial output distance function model, there is a simple way to check whether monotonicity conditions are satisfied, by looking at the signs of the estimates of b_1, b_2, and b_3. Specifically, following O'Donnell and Coelli (2005), the sample data is deflated prior to the estimation of (18) so that all output and input variables have a sample mean of one and the time trend has a sample mean of zero. When evaluated at these variable means, $\partial \ln D_o(\boldsymbol{y}, \boldsymbol{x}, t) / \partial \ln x_n$ collapses to b_n and the monotonicity conditions can therefore be expressed as $b_n \leq 0$ [see (19)]. As can be seen from Table 1.1, the estimates of b_1, b_2, and b_3 all have the right signs (i.e., $b_n \leq 0$, $i = 1, 2, 3$), implying the theoretical monotonicity constraints are satisfied.

We also calculate simulation inefficiency factor (SIF) values for all the parameters of the two regularity-constrained models. As can be seen from the last column of Table 1.1 and that of Table 1.2, all the SIF values are less than 20 and most of them are lower than 15, a quite strong indication of the convergence of the two samplers. Thus, in what follows we focus on empirical results from the two regularity-constrained models.

6.1 Productivity Growth

With the estimated parameters from the regularity-constrained translog radial output distance function, we compute average annual productivity growth over the sample period for each country, using the conventional Feng and Serletis (2010) productivity index, PG^S. The results are shown in Panel A of Table 2. As can be seen, this measure ranges from 0.0260 to 0.0525 with Ireland, Denmark, and Finland being the three top performers and Japan, the United State, and Canada being the three bottom performers. This finding is generally consistent with those found in previous studies that also employ conventional-type productivity indexes. For example, OECD (2001) applies the Jorgenson and Griliches (1967) Divisia productivity index to a group of OECD countries over the period 1980-1999 and finds that average annual productivity growth ranges from 0.0100 to 0.0400 with Ireland, Finland, and Norway being the three top performers and the U.S. and Canada being the two bottom performers.

With the estimated parameters from the regularity-constrained quadratic directional output distance function, we also compute average annual productivity growth over the sample period for each country, using the Divisia–Luenberger productivity index, PG^L. The results are shown in Panel B of Table 2. In this case, annual productivity growth ranges from 0.0223 to 0.0413, with Japan and Ireland being the two top performers and the United States and Australia being the two bottom performers.

An important question we need to address here is whether the conventional Feng and Serletis (2010) productivity index, by failing to take into account the bad output, CO_2, leads to misleading results regarding productivity growth. It is tempting to answer this question by directly comparing the magnitudes of the estimates of the conventional Feng and Serletis (2010) productivity index with those of the Divisia–Luenberger productivity index. However, this practice is inappropriate because productivity growth is measured along different directions in these two indexes. Specifically, in the case of the Divisia–Luenberger productivity index, productivity growth is measured along the direction $\boldsymbol{g} = (1, -1)$, whereas in the case of the conventional Feng and Serletis (2010) productivity index it is measured along the direction $\boldsymbol{g}(t) = (\boldsymbol{y}(t), \boldsymbol{x}(t))$. The use of different directions means that the magnitudes of the estimates of the two indexes cannot be compared directly.

To answer the above question, we instead compare the productivity ranking of individual countries based on the Divisia–Luenberger productivity index with that based on the conventional Feng and Serletis (2010) productivity index. Our logic here is that if the conventional Feng and Serletis (2010) productivity index is valid for situations where bad outputs are present, then these two indexes should yield (roughly) consistent rankings of individual countries; for example, a country that is highly (poorly) ranked according to the Divisia–Luenberger productivity index should also be highly (poorly) ranked according to the conventional Feng and Serletis (2010) productivity index. A comparison of the productivity growth estimates in Panel A of Table 2 with their corresponding estimates in Panel B of Table 2 reveals that this is not the case. For example, Japan, which ranks at the bottom according to the conventional primal Divisia productivity index, outperforms all the other countries according to the Divisia–Luenberger productivity index. France, which ranks 12th according to the conventional Feng and Serletis (2010) productivity index, ranks 5th according to the Divisia–Luenberger productivity index.

To formally examine whether the conventional Feng and Serletis (2010) productivity index yields a ranking that is roughly consistent with that yielded by the Divisia–Luenberger productivity index, we formally calculate the Spearman rank correlation coefficient between the two indexes for each of the sample years. Specifically, we first use the following posterior probability to rank individual countries

$$\text{Prob}\left(\text{Rank}\left(PG_j\right) = i\text{th}\right),\ i, j = 1, \cdots, K,$$

where K is the number of countries, i represents a particular rank, PG_j is the productivity growth of country j, and $\text{Rank}(PG_j)$ is the rank of country j in terms of productivity growth. As noted by Atkinson and Dorfman (2005), Griffiths and O'Donnell (2005), and Ntzoufras (2009), when productivity growth (or other ranking criteria) is estimated within a Bayesian framework, it is more appropriate to use posterior probabilities to rank individual countries (or firms) than to use posterior means. This is because the former approach takes into account the simulating variation of productivity growth. Since PG_j can be computed by using either the conventional Feng and Serletis (2010) productivity index or the Divisia–Luenberger productivity index, we end up with two ranking results (ranking lists): Rank_{j1}, the rank of country j based on the conventional Feng and Serletis (2010) productivity index, and Rank_{j2}, the rank of the same country based on the Divisia–Luenberger productivity index. We then calculate the Spearman rank correlation coefficient between Rank_{j1} and Rank_{j2} for each year, as follows

$$\rho = 1 - \frac{6\sum_{j=1}^{K}(\text{Rank}_{j1} - \text{Rank}_{j2})^2}{K(K^2-1)}.$$

If $\rho = -1$, there is perfect negative correlation; if $\rho = 1$, there is perfect positive correlation; and if $\rho = 0$, there is no correlation. We also calculate confidence intervals for the Spearman rank correlation coefficients. Since Rank_{j1} and Rank_{j2} are computed from two MCMC chains using different datasets, these confidence intervals cannot be obtained within a Bayesian framework. In this paper, we instead bootstrap each of the Spearman rank correlation coefficient 10,000 times.

The Spearman rank correlation coefficients and 95% bootstrap confidence intervals are presented in Table 3. As can be seen in this table, all of the Spearman rank correlation coefficients are smaller than one (ranging from 0.0750 to 0.8929) and none of the bootstrap confidence intervals contain one. Particularly, in 1983 and 1984 the Spearman rank correlation coefficient is as low as 0.0750 and 0.2214 respectively, suggesting that there is little correlation between the two rankings in these years. These results show that failure to allow for bad outputs can greatly change the ranking of individual countries and thus the conventional Feng and Serletis (2010) productivity index is not suitable for situations where undesirable outputs are present.

6.2 Technological Change and Efficiency Change

We now turn to the technological change and efficiency change components of the two indexes. The two components of the conventional Feng and Serletis (2010) productivity index, obtained by using (12), are presented in Panel A of Table 4. A comparison of these two components reveals that technological change (TC^S) is much larger than efficiency change (EC^S) for all the sample countries. Taking the United State for example, it has an average annual technological change of 0.0280, compared with an average annual efficiency change of only 0.0017. The dominance of technological change suggests that when the good output (i.e., GDP) alone is taken into account, innovation plays a much more important role in driving the productivity growth than improvements in efficiency. This finding is consistent with those from previous studies, including the classic work by Färe *et al.* (1994).

The two components of the Divisia–Luenberger productivity index, obtained by using (3), are shown in Panel B of Table 4. As with the case of the conventional Feng and Serletis (2010) productivity index, technological change (TC^L) is still the dominant force behind productivity growth. Taking the United State for example, its average annual technological change is 0.0226, whereas its efficiency change (EC^L) is only -0.0002. This suggests that even when the bad output is taken into account, innovation still plays a much more important role in driving productivity growth in the sample countries. This finding is consistent with that of Mahlberg and Sahoo (2011), who investigate the productivity growth in 22 OCED countries by estimating the directional input distance function using the data envelopment analysis (DEA) approach.

For the same reason as discussed above in the context of productivity growth, we do not directly compare the magnitudes of the TC^S and TC^L. Instead we compute the Spearman rank correlation coefficient between the ranking of individual countries based on TC^S and that based on TC^L. As can be seen from Table 5, all of the Spearman rank correlation coefficients are smaller than one and none of the bootstrap confidence intervals contain one. Particularly, the point estimates of the Spearman rank correlation coefficients from 1982 to 1988 are all negative (ranging from -0.4714 to -0.0643), suggesting that in these years there is a negative correlation between the two rankings; i.e., countries, which are highly (poorly) ranked according to the Divisia–Luenberger productivity index, are very likely to be mistakenly ranked low (high) according to the conventional Feng and Serletis (2010) productivity index. This result confirms that the use of the conventional Feng and Serletis (2010) productivity index can lead to misleading conclusions when undesirable outputs are present.

Turing to efficiency change, recall that positive values imply that the country is catching up with the best practice frontier and that negative values imply the country is lagging behind the frontier. Looking first at the efficiency change component of the Divisia–Luenberger productivity index in Panel B of Table 4, we see that only two countries, France and Ireland, show positive values, suggesting that when the bad output is taken into account, on average only these two countries are moving towards the frontier. However, when the conventional Feng and Serletis (2010) productivity index is used (see the second column of Panel A of Table 4), seven more countries (besides France and Ireland) show positive values, implying that on average these seven countries are mistakenly classified as ones whose efficiencies improve over time. This suggests that by failing to take into account the bad output, the conventional Feng and Serletis (2010) productivity index not only leads to misleading conclusions regarding technological change, but also results in wrong conclusions concerning efficiency change.

7 Conclusion

Conventional Divisia-type productivity indexes [the Solow (1957) index, the Jorgenson and Griliches (1967) index, the Fuss (1994) index, and the Diewert and Fox (2008) index, and the Divisia–Luenberger productivity index] have long enjoyed great popularity. However, they all ignore undesirable outputs. In an attempt to fill in this gap, we propose a primal Divisia-type productivity index by totally differentiating the directional output distance function with respect to time. We refer to this new productivity index as the Divisia–Luenberger productivity index. This index inherits the desirable properties possessed by the directional output distance function and allows for the simultaneous expansion of desirable outputs and contraction of undesirable outputs. We also show that the Divisia–Luenberger productivity index can be decomposed into two components: a directional-output-distance-function-based technological change term and a directional-output-distance-function-based efficiency change term. This decomposition is consistent with the tradition of Färe *et al.* (1994).

We also empirically examine the effects of failure to take into account bad outputs when they are present. This is done by comparing the Divisia–Luenberger productivity index and the conventional Divisia-type productivity indexes, using aggregate data on 15 OECD countries over the period 1981-2000. In doing so, we first choose the Feng and Serletis (2010) productivity index as a representative of the aforementioned conventional Divisia productivity indexes. This is because as theoretically shown by Feng and Serletis (2010), the Feng and Serletis (2010) productivity index is dual to all the famous Divisia-type productivity/technical change indexes in the literature. To construct the Divisia–Luenberger productivity index and the Feng and Serletis (2010) productivity index for the sample countries, we estimate a quadratic directional output distance function and a translog radial output distance function, respectively, both subject to the theoretical regularity conditions. This is done by using a Bayesian approach due to its capability of imposing nonlinear constraints. Our empirical results show that by failing to take into account the bad output, the conventional Feng and Serletis (2010) productivity index not only leads to misleading conclusions regarding productivity growth and technological change, but also results in wrong conclusions concerning efficiency change.

8 Appendix

In our particular case where there is one good output and one bad output, the left hand side of equation (1) can be expanded as follows (by equation (20))

$$
\begin{aligned}
&\vec{D}_o\left(y+kg_y, b-kg_b, \boldsymbol{x}, t; \boldsymbol{g}\right)\\
&=\beta_0+\alpha_1\left(y+kg_y\right)+\beta_1(b-kg_b)+\sum_{n=1}^{3}\gamma_n x_n+\beta_\tau t\\
&+\frac{1}{2}\alpha_{11}(y+kg_y)^2+\frac{1}{2}\beta_{11}(b-kg_b)^2+\frac{1}{2}\sum_{n=1}^{3}\sum_{n'=1}^{3}\gamma_{nn'}x_n x_{n'}+\frac{1}{2}\beta_{\tau\tau}t^2\\
&+\sum_{n=1}^{3}\delta_{n1}x_n(y+kg_y)+\sum_{n=1}^{3}\varphi_{n1}x_n(b-kg_b)+\phi_{11}(y+kg_y)(b-kg_b)\\
&+\alpha_{\tau 1}t(y+kg_y)+\beta_{\tau 1}t(b-kg_b)+\sum_{n=1}^{3}\gamma_{\tau n}t x_n\\
&=\beta_0+\alpha_1 y+\beta_1 b+\sum_{n=1}^{3}\gamma_n x_n+\beta_\tau t+\frac{1}{2}\alpha_{11}y^2+\frac{1}{2}\beta_{11}b^2+\frac{1}{2}\sum_{n=1}^{3}\sum_{n'=1}^{3}\gamma_{nn'}x_n x_{n'}+\frac{1}{2}\beta_{\tau\tau}t^2\\
&+\sum_{n=1}^{3}\delta_{n1}x_n y+\sum_{n=1}^{3}\varphi_{n1}x_n b+\phi_{11}yb+\alpha_{\tau 1}ty+\beta_{\tau 1}tb+\sum_{n=1}^{3}\gamma_{\tau n}t x_n\\
&+\Bigg[\alpha_1 kg_y-\beta_1 kg_b+\frac{1}{2}\alpha_{11}\left(2ykg_y+k^2g_y^2\right)\\
&+\frac{1}{2}\beta_{11}\left(-2bkg_b+k^2g_b^2\right)+\sum_{n=1}^{3}\delta_{n1}x_n kg_y-\sum_{n=1}^{3}\varphi_{n1}x_n kg_b\\
&+\phi_{11}\left(-ykg_b+bkg_y-k^2g_yg_b\right)+\alpha_{\tau 1}tkg_y-\beta_{\tau 1}tkg_b\Bigg]
\end{aligned}
\tag{A1}
$$

Noting that the terms in the first two lines on the right hand side of (A1) are just $\vec{D}_o\left(y, b, \boldsymbol{x}, t; \boldsymbol{g}\right)$, (A1) can be rewritten as

$$
\begin{aligned}
\vec{D}_o\left(y+kg_y, b-kg_b, \boldsymbol{x}, t; \boldsymbol{g}\right)=\vec{D}_o\left(y, b, \boldsymbol{x}, t; \boldsymbol{g}\right)+\Bigg[&\alpha_1 kg_y-\beta_1 kg_b+\frac{1}{2}\alpha_{11}\left(2ykg_y+k^2g_y^2\right)\\
&+\frac{1}{2}\beta_{11}\left(-2bkg_b+k^2g_b^2\right)+\sum_{n=1}^{3}\delta_{n1}x_n kg_y-\sum_{n=1}^{3}\varphi_{n1}x_n kg_b\\
&+\phi_{11}\left(-ykg_b+bkg_y-k^2g_yg_b\right)+\alpha_{\tau 1}tkg_y-\beta_{\tau 1}tkg_b\Bigg].
\end{aligned}
\tag{A2}
$$

According to the translation property in equation (1), the expression inside the square bracket on the right hand side of (A2) is just $-k$, that is

$$\begin{aligned}
&\alpha_1 k g_y - \beta_1 k g_b + \frac{1}{2}\alpha_{11}\left(2ykg_y + k^2 g_y^2\right) + \frac{1}{2}\beta_{11}\left(-2bkg_b + k^2 g_b^2\right) + \sum_{n=1}^{3}\delta_{n1} x_n k g_y \\
&- \sum_{n=1}^{3}\varphi_{n1} x_n k g_b + \phi_{11}\left(-ykg_b + bkg_y - k^2 g_y g_b\right) + \alpha_{\tau 1} t k g_y - \beta_{\tau 1} t k g_b \\
&= -k
\end{aligned}$$

which, after dividing both sides by k, can be rewritten as

$$\begin{aligned}
&\alpha_1 g_y - \beta_1 g_b + \frac{1}{2}\alpha_{11}\left(2yg_y + kg_y^2\right) + \frac{1}{2}\beta_{11}\left(-2bg_b + kg_b^2\right) + \sum_{n=1}^{3}\delta_{n1} x_n g_y \\
&- \sum_{n=1}^{3}\varphi_{n1} x_n g_b + \phi_{11}\left(-yg_b + bg_y - kg_y g_b\right) + \alpha_{\tau 1} t g_y - \beta_{\tau 1} t g_b \\
&= -1.
\end{aligned} \tag{A3}$$

When $(g_y, -g_b) = (1, -1)$, (A3) can be further written as

$$\alpha_1 - \beta_1 + \frac{1}{2}\alpha_{11}(2y + k) + \frac{1}{2}\beta_{11}(-2b + k) + \sum_{n=1}^{3}\delta_{n1} x_n - \sum_{n=1}^{3}\varphi_{n1} x_n + \phi_{11}(-y + b - k) + \alpha_{\tau 1} t - \beta_{\tau 1} t = -1$$

which, after rearranging, can be written as

$$\begin{aligned}
&(\alpha_1 - \beta_1 + 1) + (\alpha_{11} - \phi_{11})\, y + (\phi_{11} - \beta_{11})\, b + \left(\frac{1}{2}\alpha_{11} + \frac{1}{2}\beta_{11} - \phi_{11}\right) k \\
&+ \left(\sum_{n=1}^{3}\delta_{n1} - \sum_{n=1}^{3}\varphi_{n1}\right) x_n + (\alpha_{\tau 1} - \beta_{\tau 1})\, t \\
&= 0.
\end{aligned} \tag{A4}$$

A sufficient condition for (A4) to hold is

$$\alpha_1 - \beta_1 = -1;\ \alpha_{11} = \phi_{11} = \beta_{11};\ \delta_{n1} = \varphi_{n1}\ \ (n = 1, 2, 3)\,;\ \text{and}\ \alpha_{\tau 1} = \beta_{\tau 1}$$

which is just (21). Thus as can be seen from the above proof, (21) is obtained by the application of the translation property of the directional output distance function.

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Table 1.1. Parameter Estimates From The Constrained Translog Output Distance Function Model

Variable	Parameter	Mean	Standard deviation	95% Credible Interval	SIF
intercept	a_0	0.3974	0.0213	$(0.3571, 0.4388)$	1.3465
$\ln x_1$	b_1	-0.3675	0.0508	$(-0.4729, -0.2747)$	16.1126
$\ln x_2$	b_2	-0.4855	0.0660	$(-0.6093, -0.3610)$	5.4063
$\ln x_3$	b_3	-0.2372	0.0380	$(-0.3135, -0.1680)$	1.3369
t	δ_τ	-0.0423	0.0029	$(-0.0477, -0.0365)$	16.0887
t^2	$\delta_{\tau\tau}$	0.0007	0.0003	$(0.0001, 0.0011)$	5.4199
$t \ln x_1$	$\widetilde{\delta}_{\tau 1}$	-0.0096	0.0028	$(-0.0154, -0.0043)$	1.3668
$t \ln x_2$	$\widetilde{\delta}_{\tau 2}$	0.0084	0.0037	$(0.0013, 0.0153)$	16.1226
$t \ln x_3$	$\widetilde{\delta}_{\tau 3}$	0.0049	0.0027	$(-0.0008, 0.0100)$	5.3863
$(\ln x_1)^2$	b_{11}	0.2001	0.1014	$(0.0132, 0.4179)$	16.0623
$\ln x_1 \ln x_2$	b_{12}	-0.0450	0.1454	$(-0.3460, 0.2139)$	5.4133
$\ln x_1 \ln x_3$	b_{13}	0.0035	0.0676	$(-0.1135, 0.1514)$	1.3538
$(\ln x_2)^2$	b_{22}	0.0493	0.2069	$(-0.3279, 0.4660)$	16.1534
$\ln x_2 \ln x_3$	b_{23}	-0.0943	0.0862	$(-0.2491, 0.0827)$	5.4344
$(\ln x_3)^2$	b_{33}	0.0309	0.0611	$(-0.0990, 0.1438)$	1.3251

Table 1.2. Parameter Estimates From The Constrained Quadratic Directional Output Distance Function

Variable	Parameter	Mean	Standard deviation	95% credible interval	SIF
intercept	β_0	0.0066	0.0069	$(-0.0065, 0.0208)$	6.6766
x_1	γ_1	0.0742	0.0238	$(0.0279, 0.1210)$	11.9020
x_2	γ_2	0.1824	0.0449	$(0.0939, 0.2719)$	10.4154
x_3	γ_3	-0.0011	0.0033	$(-0.0075, 0.0054)$	10.2953
$\widetilde{b}$	β_1	0.1873	0.0626	$(0.0623, 0.3085)$	14.7051
t	β_τ	-0.0025	0.0013	$(-0.0053, 0.0002)$	8.3081
$\widetilde{b}^2$	β_{11}	-0.2709	0.0863	$(-0.4416, -0.1005)$	14.6019
t^2	$\beta_{\tau\tau}$	0.0002	0.0001	$(0.0000, 0.0005)$	9.1657
bx_1	φ_{11}	-0.2441	0.0376	$(-0.3177, -0.1693)$	12.3627
bx_2	φ_{21}	0.4753	0.0921	$(0.2926, 0.6562)$	15.2927
bx_3	φ_{31}	0.0414	0.0091	$(0.0235, 0.0593)$	10.6562
tx_1	$\gamma_{\tau 1}$	0.0145	0.0017	$(0.0111, 0.0180)$	15.5724
tx_2	$\gamma_{\tau 2}$	0.0095	0.0031	$(0.0032, 0.0156)$	10.2267
tx_3	$\gamma_{\tau 3}$	0.0000	0.0002	$(-0.0005, 0.0003)$	5.8618
$\widetilde{b}t$	$\beta_{\tau 1}$	-0.0105	0.0032	$(-0.0169, -0.0041)$	9.8852
$(x_1)^2$	γ_{11}	-0.1249	0.0196	$(-0.1637, -0.0866)$	14.0788
x_1x_2	γ_{12}	0.2953	0.0341	$(0.2276, 0.3615)$	15.2578
x_1x_3	γ_{13}	0.0277	0.0044	$(0.0188, 0.0364)$	9.4963
$(x_2)^2$	γ_{22}	-0.7529	0.0597	$(-0.8697, -0.6353)$	16.7804
x_2x_3	γ_{23}	-0.0533	0.0106	$(-0.0742, -0.0324)$	13.0239
$(x_3)^2$	γ_{33}	-0.0057	0.0010	$(-0.0078, -0.0036)$	8.4077

Table 2. Productivity Growth Indexes

Country	A. Feng and Serletis (2010)		B. Divisia–Luenberger	
	Mean	95% credible interval	Mean	95% credible interval
Australia	0.0361	(0.0318, 0.0405)	0.0238	(0.0166, 0.0316)
Canada	0.0339	(0.0287, 0.0387)	0.0288	(0.0240, 0.0336)
Denmark	0.0450	(0.0416, 0.0486)	0.0327	(0.0187, 0.0492)
Finland	0.0433	(0.0378, 0.0485)	0.0322	(0.0162, 0.0507)
France	0.0360	(0.0324, 0.0397)	0.0358	(0.0327, 0.0361)
Germany	0.0369	(0.0315, 0.0422)	0.0301	(0.0270, 0.0334)
Ireland	0.0525	(0.0479, 0.0571)	0.0384	(0.0129, 0.0743)
Italy	0.0370	(0.0333, 0.0408)	0.0350	(0.0314, 0.0386)
Japan	0.0260	(0.0186, 0.0347)	0.0413	(0.0375, 0.0450)
Netherlands	0.0375	(0.0338, 0.0410)	0.0279	(0.0208, 0.0357)
Norway	0.0380	(0.0334, 0.0422)	0.0362	(0.0223, 0.0534)
Spain	0.0392	(0.0344, 0.0443)	0.0316	(0.0272, 0.0363)
Sweden	0.0415	(0.0371, 0.0460)	0.0347	(0.0259, 0.0457)
U.K.	0.0395	(0.0352, 0.0439)	0.0388	(0.0343, 0.0434)
U.S.	0.0294	(0.0240, 0.0348)	0.0223	(0.0187, 0.0260)

Table 3. Spearman Rank
Correlation Coefficients of Productivity Growth

Year	Coefficient, ρ	95% bootstrap confidence interval
1982	0.5607	$(0.0335, 0.8137)$
1983	0.0750	$(-0.3967, 0.5210)$
1984	0.2214	$(-0.2654, 0.6478)$
1985	0.4357	$(-0.0882, 0.8180)$
1986	0.6429	$(0.2741, 0.8347)$
1987	0.5500	$(0.1686, 0.7629)$
1988	0.7179	$(0.4248, 0.8625)$
1989	0.6679	$(0.3449, 0.8296)$
1990	0.7036	$(0.2420, 0.8929)$
1991	0.6107	$(0.0108, 0.9051)$
1992	0.3929	$(-0.2199, 0.7735)$
1993	0.5571	$(-0.0664, 0.8609)$
1994	0.8929	$(0.7553, 0.9486)$
1995	0.6964	$(0.2557, 0.8852)$
1996	0.8286	$(0.5517, 0.9744)$
1997	0.8893	$(0.7091, 0.9550)$
1998	0.7179	$(0.2949, 0.9313)$
1999	0.7000	$(0.1709, 0.9388)$
2000	0.8536	$(0.6866, 0.9287)$

Table 4. Technological Change and Efficiency Change

	A. Components of Feng and Serletis (2010) index				B. Components of Divisia–Luenberger index			
	Technological Change		Efficiency Change		Technological Change		Efficiency Change	
Country	Mean	95% credible interval	Mean	95% credible interval	Mean	95% credible interval	Mean	95% credible interval
Australia	0.0382	$(0.0364, 0.0399)$	-0.0016	$(-0.0060, 0.0026)$	0.0295	$(0.0256, 0.0334)$	-0.0057	$(-0.0120, 0.0012)$
Canada	0.0370	$(0.0318, 0.0429)$	-0.0025	$(-0.0076, 0.0013)$	0.0293	$(0.0245, 0.0339)$	-0.0005	$(-0.0028, 0.0031)$
Denmark	0.0453	$(0.0431, 0.0475)$	0.0001	$(-0.0028, 0.0032)$	0.0330	$(0.0255, 0.0406)$	-0.0003	$(-0.0137, 0.0164)$
Finland	0.0428	$(0.0404, 0.0449)$	0.0008	$(-0.0043, 0.0060)$	0.0349	$(0.0277, 0.0420)$	-0.0027	$(-0.0187, 0.0162)$
France	0.0358	$(0.0338, 0.0378)$	0.0005	$(-0.0024, 0.0038)$	0.0354	$(0.0324, 0.0384)$	0.0005	$(-0.0007, 0.0023)$
Germany	0.0339	$(0.0315, 0.0364)$	0.0031	$(-0.0019, 0.0082)$	0.0343	$(0.0312, 0.0376)$	-0.0042	$(-0.0067, -0.0018)$
Ireland	0.0516	$(0.0482, 0.0550)$	0.0012	$(-0.0013, 0.0047)$	0.0329	$(0.0154, 0.0526)$	0.0056	$(-0.0175, 0.0395)$
Italy	0.0369	$(0.0344, 0.0397)$	0.0004	$(-0.0025, 0.0037)$	0.0355	$(0.0323, 0.0387)$	-0.0005	$(-0.0022, 0.0017)$
Japan	0.0332	$(0.0297, 0.0369)$	-0.0063	$(-0.0119, 0.0007)$	0.0435	$(0.0397, 0.0474)$	-0.0022	$(-0.0032, -0.0011)$
Netherlands	0.0389	$(0.0370, 0.0409)$	-0.0010	$(-0.0047, 0.0023)$	0.0294	$(0.0253, 0.0335)$	-0.0015	$(-0.0077, 0.0059)$
Norway	0.0376	$(0.0334, 0.0415)$	0.0007	$(-0.0012, 0.0036)$	0.0361	$(0.0267, 0.0457)$	0.0000	$(-0.0121, 0.0169)$
Spain	0.0424	$(0.0401, 0.0444)$	-0.0025	$(-0.0072, 0.0021)$	0.0397	$(0.0355, 0.0439)$	-0.0081	$(-0.0117, -0.0035)$
Sweden	0.0426	$(0.0397, 0.0460)$	-0.0007	$(-0.0049, 0.0035)$	0.0365	$(0.0319, 0.0411)$	-0.0017	$(-0.0104, 0.0091)$
U.K.	0.0394	$(0.0366, 0.0419)$	0.0005	$(-0.0031, 0.0044)$	0.0389	$(0.0343, 0.0436)$	0.0000	$(-0.0015, 0.0021)$
U.S.	0.0280	$(0.0237, 0.0324)$	0.0017	$(-0.0018, 0.0059)$	0.0226	$(0.0191, 0.0262)$	-0.0002	$(-0.0008, 0.0004)$

Table 5. Spearman Rank Correlation Coefficients of Technological Change

Year	Coefficient, ρ	95% bootstrap confidence interval
1982	-0.4464	$(-0.7625, -0.0059)$
1983	-0.4714	$(-0.7660, -0.0312)$
1984	-0.3393	$(-0.7121, 0.1908)$
1985	-0.3071	$(-0.7155, 0.2913)$
1986	-0.2857	$(-0.7189, 0.3318)$
1987	-0.2321	$(-0.6687, 0.3526)$
1988	-0.0643	$(-0.5732, 0.4766)$
1989	0.0571	$(-0.4080, 0.5020)$
1990	0.3179	$(-0.2203, 0.7199)$
1991	0.4821	$(-0.0044, 0.8080)$
1992	0.4964	$(0.0029, 0.8146)$
1993	0.5000	$(0.0143, 0.8209)$
1994	0.5036	$(0.0056, 0.8192)$
1995	0.5429	$(0.0920, 0.8260)$
1996	0.6000	$(0.1329, 0.8466)$
1997	0.6000	$(0.1328, 0.8448)$
1998	0.6321	$(0.1711, 0.8698)$
1999	0.7000	$(0.2980, 0.8972)$
2000	0.7071	$(0.3039, 0.9055)$